INFLUENCE OF CONTROLLABILITY-TYPE PROPERTIES ON RELIABILITY AND STABILITY OF DESYNCHRONIZED SYSTEMS

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Abstract: Systems with high level of failures are considered that permanently operate due to this reason in a transient dynamical regime. In real systems amplitude restrictions for state variables are inevitable and also the so called overshooting effect is inwardly intrinsic. Therefore the operation in transient regime can lead to a great number of secondary failures. At the same time it is known that some classes of the so called desynchronized systems are insensitive to failures in data communication links. In the paper the notion of quasi-controllability for desynchronized systems is introduced that allow easy and efficient to estimate the overshooting measure. This presents the means to design the variety of highly reliable fault-tolerant control and manufacturing systems.

INTRODUCTION

The realistic approach to the problem of reliability and stability of multicomponent manufacturing control systems has to take into account a variety of properties of these systems. Among them are the possibility of malfunctioning of particular links, asynchronous way of operation of control devices (e.g., processors) and so on. Traditionally, the reliability analysis of complex real systems is based on probabilistic reasoning. Under these circumstances it is assumed, as a rule, that

- failures in devices are reasonable rare and are independent one from another,
- the greater part of the operational time the system works in a steady state regime.

However, in the last time many examples appear when the difficulties of probabilistic analysis of complex systems functioning grow to such an extent that no practical conclusions for realistic situations are possible. Often the behavior of such systems is characterized by the following features:

- the level of failures or other kind of malfunctions is very high and often faults correlate one with another,
- in virtue of high frequency of failures the system permanently operates within the transient regime,
- in virtue of transient operational regime, high amplitudes of the state vector are possible in the system that can lead to secondary failures because of overshooting of parameters beyond admissible bounds.

Examples of such a kind of systems are complex flexible industries, man — computer systems, systems with microprocessor controlled devices, large systems that controlled via computer networks, and so on (see, e.g., (Bertsekas and Tsitsiklis, 1988)). One way to overcome difficulties mentioned above is to develop a special hardware. Another way is to give an adequate mathematical description of corresponding phenomena. The need of latter forced to appear within last years a new chapter of the control theory — the theory of the so called desynchronized systems (see, e.g., (Kleptsyn *at al.*, 1984); (Bertsekas and Tsitsiklis, 1988); (Asarin *at al.*, 1988, 1990); (Krasnoselskii *at al.*, 1991)). The aim of this paper is to propose some new efficient method to estimate reliability of desynchronized systems.

DESYNCHRONIZED AND FAULT-TOLERANT SYSTEMS

Consider a system S consisting of N interconnected subsystems $S_1, S_2, \ldots, S_N$, that interact in some discrete time instants $\{T^n\}$, $-\infty < n < \infty$. Depending on a problem formulation, on internal characteristics of the system and other factors the moments of interacting can be influenced by some deterministic or stochastic law. They can depend also from the internal state of the system. Denote the value of the state vector of the subsystem S_i on the interval $[T^n, T^{n+1})$ by $x_i(n)$, $-\infty < n < \infty$. It is supposed that the vector $x_i(n)$ takes values in some finite-dimensional space $\mathbb{R}^{d_i}$, where $d_i \geq 1$. In this case the state space $\mathcal{X}$ of the system S can be identified with the Cartesian product $\mathcal{X} = \mathbb{R}^{d_1} \times \mathbb{R}^{d_2} \times \cdots \times \mathbb{R}^{d_N}$. It is convenient to denote the state vector $x \in \mathcal{X}$ of the system S by $x = \{x_1, x_2, \ldots, x_N\}$, where $x_i \in \mathbb{R}^{d_i}$, $i = 1, 2, \ldots, N$. In what follows it will be supposed that the updating of the state vector of the system S happens infinitely often, i.e., $T^n \to \infty$ when $n \to \infty$.

In the idealistic case, when the system S is absolutely reliable and there are no failures in it, the change of its state at a moment T^{n+1} is fulfilled according to the functional law

$$x_{\text{new}} = f(x_{\text{old}}). \tag{1}$$

Here $f : \mathcal{X} \mapsto \mathcal{X}$ is some function depending on the dynamic characteristics of the system, x_{old} is the state vector of the system immediately before the updating moment T^{n+1}, and x_{new} is the state vector of the system immediately after the updating moment T^{n+1}. Denote $x(n) = x_{\text{old}}$, $x(n+1) = x_{\text{new}}$. Then for the case of the free-of-faults system S the following dynamics equation can be written

$$x(n+1) = f(x(n)), \qquad -\infty < n < \infty. \tag{2}$$

Now, let us formulate the basic assumption:

let at any moment $T^n \in \{T^k : \quad -\infty < k < \infty\}$ only few of the subsystems of the system S update their states following the law (1) — let it will be the subsystems S_i with the indexes from some index set $\omega(n) \subseteq \{1, 2, \ldots, N\}$; and let the subsystems S_j, $j \in \{1, 2, \ldots, N\} \setminus \omega(n)$, for a variety of reasons do not update their state

To describe the dynamics of the system S in this situation it is convenient to use the coordinate representation of vectors and functions. Let us introduce the following designations

$$\begin{aligned}
x_{\text{old}} &= \{x_{\text{old},1}, x_{\text{old},2}, \ldots, x_{\text{old},N}\},\\
x_{\text{new}} &= \{x_{\text{new},1}, x_{\text{new},2}, \ldots, x_{\text{new},N}\},\\
x(n) &= \{x_1(n), x_2(n), \ldots, x_N(n)\},\\
x(n+1) &= \{x_1(n+1), x_2(n+1), \ldots, x_N(n+1)\},\\
f(x) &= \{f_1(x_1, x_2, \ldots, x_N), f_2(x_1, x_2, \ldots, x_N), \ \ldots \ , f_N(x_1, x_2, \ldots, x_N)\},\\
f_i &: \mathbb{R}^{d_1} \times \mathbb{R}^{d_2} \times \ldots \times \mathbb{R}^{d_N} \mapsto \mathbb{R}^{d_i}, \qquad i = 1, 2, \ldots, N.
\end{aligned}$$

Then the law of the state vector updating for the system S in coordinate notation takes the form

$$x_i(n+1) = \begin{cases} f_i(x_1(n), x_2(n), \ldots, x_N(n)), & \text{if } i \in \omega(n),\\ x_i(n), & \text{if } i \notin \omega(n). \end{cases}$$

Introduce for every set $\omega \subseteq \{1, 2, \ldots, N\}$ the auxiliary mapping (ω-*mixture* of the mapping f):

$$f_\omega(x) = \begin{cases} f_i(x_1, x_2, \ldots, x_N), & \text{if } i \in \omega,\\ x_i, & \text{if } i \notin \omega. \end{cases}$$

Then the dynamics equation for the system S under the presence of failures can be written in the following compact form:

$$x(n+1) = f_{\omega(n)}(x(n)), \qquad -\infty < n < \infty. \tag{3}$$

In the paper for the sake of simplicity the main attention is paid to the case when the mapping $f(\cdot)$ is linear, i.e., its components $f_i(\cdot)$, $i = 1, 2, \ldots, N$, are defined by the equations

$$f_i(x_1, x_2, \ldots, x_N) = \sum_{i=1}^{N} a_{ij} x_j,$$

where a_{ij}, $i, j = 1, 2, \ldots, N$, are scalar entries. Note that the mapping f is uniquely defined by the matrix $A = (a_{ij})$. To obtain the matrix A_ω of the mapping f_ω it is sufficient to replace the strings of A with the indexes $i \notin \omega$ by corresponding strings of the identity matrix I.

Example 1. *Let the set ω has a single element, say $\omega = \{i\}$, $i = 1, 2, \ldots, N$. Then the matrix A_ω takes the form*

$$A_\omega = \begin{pmatrix} 1 & 0 & \ldots & 0 & \ldots & 0\\ 0 & 1 & \ldots & 0 & \ldots & 0\\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots\\ a_{i1} & a_{i2} & \ldots & a_{ii} & \ldots & a_{iN}\\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots\\ 0 & 0 & \ldots & 0 & \ldots & 1 \end{pmatrix}. \tag{4}$$

The dynamics of the linear system S under the presence of failures is described by the equation

$$x(n+1) = A_{\omega(n)}x(n), \qquad -\infty < n < \infty. \tag{5}$$

Describe another situation in which the dynamics of some system is covered by the equation (3). Consider a system S the state of which can be influenced by M *managers* (controllers, processors etc.) at some discrete time instants (*updating moments*). Let the set $\mathcal{T}$ of all the updating moments can be partitioned into M disjoint subsets $\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_M$, with the rule of correction of the state of S at the instants in $\mathcal{T}_i$ depending on the wishes of the i-th manager. For any updating moment $t \in \mathcal{T}$ denote by $\omega(t)$ the set of numbers of the managers governing the system at the instant t. If the wishes of the managers are the same and they implement them reliably, then the dynamics of the system S can be described in a traditional way — in terms of the difference equation (2). In the opposite case the situation is more complicated. Such systems are referred to as *desynchronized* ones. Assume that the state of S is described by the vector $x = \{x_1, x_2, \ldots, x_N\}$ in the space $\mathbb{R}^N$ that is provided with the norm $\|x\| = |x_1| + |x_2| + \cdots + |x_N|$. A function $x(t)$ corresponding to an admissible dynamics of the system S is by definition piecewise constant and all its points of discontinuity belong to the set $\mathcal{T}$. The system S is said to be *linear desynchronized* if such matrixes A_m $(m = 1, 2, \ldots, M)$ can be chosen that at any instant $t_* \in \mathcal{T}_m$ $(m = 1, 2, \ldots, M)$ the following equality is true

$$x(t_* + 0) = A_m x(t_* - 0).$$

Denote by $x(t) = S(t; \mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_M)$ the output of the system S corresponding to the initial state $x_0 = x(0)$ and to the collection of the sets of updating moments corresponding to $\mathcal{T}_1, \mathcal{T}_2, \ldots, \mathcal{T}_M$. Let us restrict the consideration to the case when the set of all possible updating moments coincides with the set $\mathbb{N}$ of integers 1,2,.... Then we shall come to conclusion that the dynamics of the system S will be described by the equation (3).

STABILITY AND OVERSHOOTING

Suppose that there are no failures in the system S. In this case stability is one of traditional and most important characteristics of the 'quality' of the investigated system. Surely the high level of failures in subsystems of the system S may cause the destruction of the stability of the system S. By this reason it is useful to distinguish the class of systems that are robust with respect to permanently high level of failures. Introduce the corresponding definition only for linear systems.

Definition 2. *A desynchronized system S, whose dynamics is described by the equation* (5) *is said to be absolutely asymptotically stable if for any sequences $\{\omega(\cdot) : \omega(n) \subseteq \{1, 2, \ldots, N\}\}$ solutions $x(n)$ of the equation* (5) *tend to equilibrium uniformly with respect to initial values $x(0)$ from the unit ball in some norm.*

Some classes of absolutely stable desynchronized systems were described and investigated in works (Kleptsyn *at al.*, 1984); (Bertsekas and Tsitsiklis, 1988); (Asarin *at al.*, 1988, 1990); (Krasnoselskii *at al.*, 1991); (Kozyakin, 1991). There was established that for any absolutely stable desynchronized system S there exists a constant $\mu < \infty$, satisfying the inequality

$$\|x(n)\| \le \mu \|x(0)\|, \qquad \forall n \ge 0, \quad \forall \{\omega(\cdot)\} : \omega(n) \subseteq \{1, 2, \ldots, N\}. \tag{6}$$

The distinguishing of systems possessing the stability property under arbitrary failures in subsystems does not solve completely the problem of reliability. Modes in stable systems have 'a good behavior at infinity'; but before that the system works a certain time in the so called transition regime. The main problem often consists in the presence of the so called 'peak' effect or 'overshooting' within the transition regime. The essence of overshooting is big (however short-lived) increasing of amplitudes of system states before these states would appear near equilibrium.

Probably, the first examples of linear systems with peaks were mentioned in the works of Bongiorno and Youla (1968) and Mita (1976). These works had attracted the attention of different scientists to the peaking effect. In (Polotskii, 1978) and (Zeitz, 1983) the effect of peaking in some systems with scalar inputs or outputs was studied. Mita and Yoshida (1980) had noted that the peaks exist in a variety of realistic cases.

The systems with big degree of stability have one important feature: these systems often have very poor robustness. The latter was noted by Soroka and Shaked (1984), Olbrot and Cieslik (1988), Lehtomaki *at al.* (1981). Therefore the necessity of explaining and investigating this effect has arisen. Partially this was done by Izmailov (1977, 1988).

Definition 3. *Overshooting measure* $\mathrm{ovm}(S)$ *of the desynchronized system is by definition the infimum of the values of* μ *satisfying the inequality* (6).

Obtaining of upper bounds for the overshooting measure $\mathrm{ovm}(S)$ is an important stage in investigation of reliability of desynchronized system.

QUASI-CONTROLLABILITY

This section contains main results that allow to estimate the overshooting for a class of desynchronized linear systems. Denote by $\mathcal{A}$ the set of matrices A of the form $\{A_\omega : \ \omega \subseteq \{1, 2, \ldots, N\}\}$. Evidently, $I \in \mathcal{A}$.

Definition 4. *System* S *is called to be quasi-controllable if there is no nontrivial proper subspace* $L \subseteq \mathbb{R}^N$ *invariant for any matrix* $M \in \mathcal{A}$.

Denote by $\mathcal{A}_k$ $(k = 1, 2, \ldots)$ the set of all finite products of the matrixes from $\mathcal{A}$ with not more then k multipliers. The set $\mathcal{A}_k(x)$ is the set of all vectors $\{Mx : \ M \in \mathcal{A}_k\}$. Denote by $\mathrm{co}(W)$, $\mathrm{absco}(W)$ and $\mathrm{span}(W)$, correspondingly, the convex hull, the absolute convex hull and the linear hull of the vector set W. Recall that the set W is absolutely convex if it is convex and with any point x contains also the point $-x$. Absolutely convex hull of W is the intersection of all absolutely convex sets containing W. Let $B(t)$ be a zero-centered ball of the radius t.

To some extent, the meaning of the quasi-controllability notion is clarified by the following definition and theorem.

Definition 5. *For any* $n \geq N$ *the n-measure of quasi-controllability of a desynchronized system* S *is by definition the number*

$$\mathrm{qcm}_n(S) = \inf_{x \in \mathbb{R}^N, \|x\|=1} \sup\{\rho : \ B(\rho) \subseteq \mathrm{absco}(\mathcal{A}_n(x))\}$$

Theorem 6. *The n-measure of quasi-controllability is positive if and only if the system* S *is quasi-controllable one.*

Let us present the main result — the theorem on the a'priori estimation of the overshooting measure.

Theorem 7. *Let the desynchronized system S be stable. Then for every $n \geq N$ it is true the inequality*

$$\operatorname{ovm}(S) \leq \frac{1}{\operatorname{qcm}_n(S)}.$$

Theorem 7 may be considered as a new type of uncertainty principle.

The last theorem is useful because often it is easy to obtain constructive lower estimate for the measure of quasi-controllability (and therefore by Theorem 7 constructive upper estimate for overshooting measure). Let us present one result of a kind. Matrix $A = (a_{ij})$ with scalar entries is called to be *irreducible* if it cannot be represented in a block triangle form by any renumeration of the basis elements in $\mathbb{R}^N$.

Theorem 8. *The linear system S with the matrix A is quasi-controllable if and only if the matrix A is irreducible and 1 is not its eigenvalue. Then the estimate $\operatorname{qcm}_N(S) \geq \alpha\kappa^{N-1}$ is true, where*

$$\kappa = \frac{1}{2}\min\{|a_{ij}| : \ i \neq j, a_{ij} \neq 0\}, \quad \alpha = \frac{1}{2N}\min\{\|(A-I)x\| : \ \|x\| = 1\}.$$

Proof. Let 1 be the eigenvalue of the matrix A. Denote by x_* corresponding eigenvector. Then x_* is an eigenvector corresponding to the eigenvalue 1 for any matrix $A_\omega \in \mathcal{A}$. So the system S is not quasi-controllable.

Suppose that the matrix A is reducible. We can suppose without loss of generality that a certain subspace $E_p = \operatorname{span}\{e_1, e_2, \ldots, e_p\}$ $(p < N)$ is invariant for the matrix A. In this case the subspace E_p is invariant for any matrix $A_\omega \in \mathcal{A}$. So the system S does not possess the property of quasi-controllability in this case as well.

Let us prove the quasi-controllability of the system S in the case when 1 is not the eigenvalue of the matrix A and the matrix A is irreducible. It is sufficient to prove for any nonzero vector $x \in \mathbb{R}^N$ the equality

$$\operatorname{span}\{\mathcal{A}_N(x)\} = \mathbb{R}^N. \tag{7}$$

Choose an arbitrary vector $x \in \mathbb{R}^N$, $\|x\| = 1$, and consider vectors $(A_1 - I)x$, $(A_2 - I)x$, $\ldots$, $(A_N - I)x \in \operatorname{span}\{\mathcal{A}_1(x)\}$. By definition of the ω-mixture of a matrix

$$(A - I)x = (A_1 - I)x + (A_2 - I)x + \ldots + (A_N - I)x,$$

and 1 is not an eigenvalue of the matrix A. So at least one of vectors $(A_1 - I)x$, $(A_2 - I)x$, $\ldots$, $(A_N - I)x$ is not equal to zero. Assume without loss of generality that $(A_1 - I)x \neq 0$ and $\|(A_1 - I)x\| \geq \frac{1}{N}\|(A - I)x\| \geq 2\alpha$. It is true the equality

$$(A_i - I)x = \langle \tilde{a}_i, x\rangle e_i, \qquad (i = 1, 2, \ldots, N), \tag{8}$$

where $\langle \cdot, \cdot \rangle$ denotes the Eucleadean scalar product in $\mathbb{R}^N$ and vectors $\tilde{a}_i$ are of the form

$$\tilde{a}_i = \{a_{i1}, a_{i2}, \ldots, a_{ii} - 1, \ldots, a_{iN}\} \qquad (i = 1, 2, \ldots, N).$$

So $\langle \tilde{a}_1, x\rangle e_1 \neq 0$, $\langle \tilde{a}_1, x\rangle e_1 \in \operatorname{span}\{\mathcal{A}_1(x)\}$ and $\|\langle \tilde{a}_1, x\rangle e_1\| \geq 2\alpha$. Consequently

$$e_1 \in \operatorname{span}\{\mathcal{A}_1(x)\} \tag{9}$$

and the vector $\frac{1}{2}\langle\tilde{a}_1, x\rangle e_1 = \frac{1}{2}A_1x - \frac{1}{2}x$ belongs to $\text{absco}\{\mathcal{A}_1(x)\} \subseteq \text{absco}\{\mathcal{A}_N(x)\}$. We got the inclusion

$$\alpha e_1 \in \text{absco}\{\mathcal{A}_N(x)\}.$$

It follows from irreducibility of the matrix A that the subspace $\text{span}\{e_1\}$ is not invariant with respect to A. In other words at least one of coordinates of the vector Ae_1, which number is not equal to 1, is nonzero. Assume without loss of generality that the second coordinate Ae_1 is not equal to zero. But the second coordinate of the vector Ae_1 coincides with the second coordinate of the vector A_2e_1 and also of the vector $(A_2 - I)e_1$. Consequently $(A_2 - I)e_1 \neq 0$. The inclusion $(A_2 - I)e_1 \in \text{span}\{\mathcal{A}_2(x)\}$ follows now from the last inequality and (9). In virtue of (8) we obtain

$$e_2 \in \text{span}\{\mathcal{A}_2(x)\}.$$

Hence the vector $\frac{1}{2}a_{21}e_2 = \frac{1}{2}\langle\tilde{a}_2, e_1\rangle e_2 = \frac{1}{2}A_2e_1 - \frac{1}{2}e_1$ belongs to $\text{absco}\{\mathcal{A}_2(e_1)\}$ and consequently to $\text{absco}\{\mathcal{A}_N(x)\}$. So

$$\alpha\kappa e_2 \in \text{absco}\{\mathcal{A}_N(x)\}.$$

Analogously, it may be concluded from irreducibility of the matrix A that following to proper renumeration of basis vectors $e_1, e_2, e_3, \ldots, e_N$ the inclusions

$$e_i \in \text{span}\{\mathcal{A}_i(x)\} \qquad (i = 1, 2, \ldots, N), \tag{10}$$

$$\alpha\kappa^{i-1}e_i \in \text{absco}\{\mathcal{A}_N(x)\}.$$

will be fulfilled. From (10) it follows the equality (7) and the estimate

$$\text{qcm}_N(S) \geq \alpha\kappa^{N-1}.$$

Theorem 8 is proved.

CONCLUSION

In the paper systems with high level of failures are considered. Due to this feature such systems permanently operate in a transient regime. That implies a possibility of amplitude overshooting of system states before these states would appear sufficiently close to equilibrium. In real systems amplitude restrictions for state variables are inevitable. Therefore the overshooting can increase the number of secondary failures.

In the paper a new approach to reliability analysis is presented that is based on the properties of the so called desynchronized systems. Thou all the facts are formulated for linear time-invariant discrete event systems, they can be easily generalized to different cases including many well known control systems. So, the idea of Theorem 7 concerning relationship between overshooting measure and quasi-controllability measure may be applied to a variety of classes of dynamic systems. This idea presents new possibilities for studying of overshooting measure even for classical control systems with computers or microprocessors in feedback links.

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